

2601/103

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ENGINEERING MATHEMATICS I

Oct./Nov. 2021

Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL

**DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)**

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Drawing instruments;

Mathematical tables/Non-programmable scientific calculator.

*This paper consists **EIGHT** questions.*

*Answer any **FIVE** questions in the answer booklet provided.*

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Determine the fifth term of the binomial expansion of $(3x + 4y)^{16}$ and evaluate its value at $x = \frac{1}{3}$ and $y = \frac{1}{4}$. (6 marks)

(b) Obtain the first three terms in the binomial expansion of $(8 - x)^{\frac{1}{3}}$. State the range of x for which the expansion is valid. (6 marks)

$$(1-x)^{\frac{1}{3}} = 1 + nx - \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!}$$

(c) The resonant frequency of a series circuit is given by $f = \frac{1}{2\pi\sqrt{LC}}$, where L is inductance, C is capacitance. Use binomial theorem to determine the approximate change in f if L increases by 1% and C decreases by 2%. (8 marks)

2. (a) Given that α and β are the roots of the equation $ax^2 + bx + c = 0$ where a , b and c are constants. Express in terms of a , b and c .

(i) $\alpha^2 + \beta^2$;

(ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.

(5 marks)

(b) Solve the equation by using formula method:

$$\frac{1}{(x-3)} + \frac{4}{(x-1)} = 2$$

(7 marks)

(c) By applying Kirchoff's law to a d.c network, the following simultaneous equations are obtained:

$$2I_1 - 3I_2 + I_3 = 4$$

$$3I_1 + 2I_2 - 2I_3 = 2$$

$$4I_1 - I_2 + 3I_3 = 16$$

Use substitution method to determine the values of the currents, correct to 2 decimal places. (8 marks)

3. (a) Simplify:

(i) $\left(\frac{1}{x} - \frac{1}{y}\right) \div \frac{(y^2 - x^2)}{x^2y^2}$

(ii) $\log_{\frac{1}{3}}8 + \log_2\left(\frac{1}{8}\right) + \log_3\left(\frac{1}{9}\right)$.

(7 marks)

(b) Solve the equations:

(i) $5(5^{\log_{10}x}) + 5^{(2-\log_{10}x)} = 30;$

(ii) $\log_4x - 2\log_x4 = 1.$

(13 marks)

4. (a) Given that $\sin A = \frac{24}{25}$ and $\cos B = -\frac{5}{13}$ where A is acute and B is obtuse. Determine:

(i) $\sin(A - B);$

(ii) $\cos(A + B).$

$(1 - \sin \theta)(1 - \cos \theta)$

$\frac{\sin^2 \theta + \frac{1}{\cos \theta}}{\cos \theta}$

(7 marks)

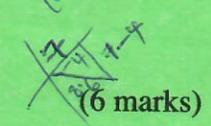
(b) Prove the identities:

(i) $\frac{1}{(1 - \sin \theta)} - \frac{1}{(1 + \sin \theta)} = 2 \tan \theta \sec \theta;$

(ii) $\frac{2 \sin 4A + \sin 6A + \sin 2A}{2 \sin 4A - \sin 6A - \sin 2A} = \cot^2 A.$

$\frac{1}{\tan \theta} = \cot \theta$
 $R \cos \alpha = 7$
 $R \sin \alpha = 4$
 $\sin \alpha = \frac{4}{R}$

$R \cos \alpha = 4$
 $\cos \alpha = \frac{4}{R}$



(6 marks)

(c) Express the function:

(i) $7 \sin \phi + 4 \cos \phi$ in the form $R \cos(\phi - \alpha)$ where $R > 0$ and $0 \leq \alpha \leq 90^\circ$.

(ii) Hence solve

$7 \sin \phi + 4 \cos \phi = \sqrt{65}$ for $0 \leq \phi \leq 360^\circ$.

$R \cos \theta \sin \alpha - R \sin \theta \cos \alpha$
 $R \sin \alpha = 4$
 $\sin \alpha = \frac{4}{R}$

(7 marks)

5. (a) Given the complex numbers

$Z_1 = 2 + 3j, Z_2 = 4 - 3j$ and $Z_3 = 1 + 4j$, express

$Z = \frac{Z_1 Z_2 Z_3}{Z_1 + Z_2 + Z_3}$ in the form $a + jb$.

$R \cos \alpha = 7$
 $\cos \alpha = \frac{7}{R}$

$R = 8.06$

(6 marks)

(b) Find all the roots of the equation $Z^3 - 1 - j\sqrt{3} = 0$ in polar form.

(6 marks)

(c) Given that $Z = -2 - j$ is a root of the equation $Z^4 + 10Z^3 + 39Z^2 + 70Z + 50 = 0$ determine the other roots.

(8 marks)

6. (a) Given that $MCosh3x + NSinh3x \equiv 7e^{3x} + 6e^{-3x}$ determine the values of M and N. (5 marks)

(b) (i) Prove that

$$Sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1});$$

(ii) Hence evaluate $Sinh^{-1}(3)$ to three decimal places. (5 marks)

(c) Show that:

(i)
$$\frac{Sinhx}{(Coshx - 1)} = Cothx + Cosechx$$

(ii)
$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$
 (10 marks)

7. (a) Find $\frac{dy}{dx}$ of $g = e^x$ from first principles. (6 marks)

(b) Given the following functions, find $\frac{dy}{dx}$

(i) $y = x^2 \cos^3(4x)$

(ii) $y = \ln\left(\frac{2x}{3x^2 + 4}\right)$

(iii) $y = \frac{(x-1)^2}{x+4}$ (9 marks)

(c) A function $z = f(x, y) = e^{xy}$, show that:

$$\frac{1}{y} \frac{\partial z}{\partial x} + \frac{1}{x} \frac{\partial z}{\partial y} = 2z$$
 (5 marks)

8. (a) Evaluate the integrals:

(i) $\int_0^{\frac{\pi}{12}} \tan^2(3x) dx$

(ii) $\int \frac{x+15}{(x-2)(x+3)} dx$

(iii) $\int_1^4 \sqrt{5x-4} dx$ (12 marks)

- (b) (i) Sketch the region bounded by $y = x^2$ and $x = y^2$ and hence find the area.
- (ii) Determine the mean value of the current function $I = 50 \sin(\omega t)$ over the interval $t = 0$ and $t = \pi$ in terms of ω and π . (8 marks)

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